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FINAL REPORT

**EXPLOITING COLLATERAL INFORMATION IN
THE ESTIMATION OF ITEM PARAMETERS**

Robert J. Mislevy

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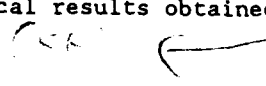
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Abstract

When using item response theory (IRT) models in educational and psychological measurement, it is standard practice to estimate the operating characteristics of test items from examinees' item responses alone. This is the final report of a project that employed Bayesian and empirical Bayesian methods to exploit additional information that is often available about test items (e.g., format, content, or cognitive processing requirements) or about examinees (e.g., educational background or demographic status). Practical and theoretical results obtained in a series of research reports are summarized.

Key words: Bayesian Estimation, Collateral Information,
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Introduction

Item response theory (IRT) models in psychometrics give the probability that an examinee will respond correctly to a given test item in terms of parameters for just that examinee and that item. This formulation makes it possible to solve many practical measurement problems that are difficult or intractable under classical test theory, including adaptive ability testing, large population equating studies, and test construction to targeted operating specifications.

It is standard practice to estimate IRT item parameters solely from the observed responses of a sample of examinees. This project was motivated by a desire to improve estimation by exploiting collateral information that is often available about test items (e.g., format, content, or cognitive processing requirements) or about examinees (e.g., educational background or demographic status). Table 1 lists the reports from the project exploring both practical and theoretical aspects of the problem. The present report summarizes the main results. The interested reader is referred to the individual papers for details, derivations, and examples.

Table 1 about here

Incorporating Collateral Information into IRT

The initial thrusts of the project were to determine how to incorporate collateral information into estimation procedures when the IRT model is correct, and to gauge its impact on estimation precision. Bayesian and empirical Bayesian methods were employed to this end. This section describes the basic model (Mislevy, 1987; in press).

Under an IRT model, the probability of response x_j to Item j with a possibly vector-valued item parameter β_j from an examinee with proficiency parameter θ is given as

$$P(x_j | \theta, \beta_j) = f(x_j | \theta, \beta_j) , \quad (1)$$

where the form of the item response function f is known up to the item parameters. Under the usual assumption of local independence, the conditional probability of the response pattern $\underline{x} = (x_1, \dots, x_n)$ to n test items is simply the product of expressions like (1):

$$P(\underline{x} | \theta, \underline{\beta}) = \prod_j P(x_j | \theta, \beta_j) , \quad (2)$$

where $\underline{\beta} = (\beta_1, \dots, \beta_n)$. Let the data matrix $\underline{X} = (\underline{x}_1, \dots, \underline{x}_N)$ represent response vectors observed from a sample of N examinees from a population in which θ follows the density $p(\theta)$. The likelihood for $\underline{\beta}$ induced by \underline{X} is obtained as

$$L_x(\underline{\beta}|\underline{X}) = \prod_i \int f(x_i|\theta, \underline{\beta}) p(\theta) d\theta . \quad (3)$$

"Marginal maximum likelihood" (MML) estimates of item parameters (e.g., Bock and Aitkin, 1981) are obtained by maximizing (3) with respect to $\underline{\beta}$.

Suppose that in addition to item responses, values of collateral variables y are also available from examinees. The appropriate marginal likelihood is now

$$L_{xy}(\underline{\beta}|\underline{X}, \underline{Y}) = \prod_i \int f(x_i|\theta, \underline{\beta}) p(\theta|y_i) d\theta . \quad (4)$$

MML estimates of item parameters that exploit collateral information about examinees are obtained by maximizing (4) with respect to $\underline{\beta}$ (Mislevy, 1987).

Bayesian item parameter estimates are obtained from posterior distributions for $\underline{\beta}$, which arise as the normalized product of a likelihood function such as (3) or (4) and a prior distribution for $\underline{\beta}$, say $g(\underline{\beta})$. If, before observing data, one possesses no information to differentiate expectations about the parameters of different items, an exchangeable prior for $\underline{\beta}$ is appropriate; that is, the items are modeled as if they were n random draws from the same distribution. In this case the posterior distribution is given by

$$p_x(\underline{\beta}|\underline{X}) \propto L_x(\underline{\beta}|\underline{X}) \prod_j g(\beta_j) \quad (5)$$

or

$$p_{xy}(\underline{\beta}|\underline{X},\underline{Y}) \propto L_{xy}(\underline{\beta}|\underline{X},\underline{Y}) \prod_j g(\beta_j) , \quad (6)$$

depending on whether collateral information is available about examinees. If values on the collateral variable z are additionally available about items, they are incorporated as

$$p_{xz}(\underline{\beta}|\underline{X},\underline{Z}) \propto L_x(\underline{\beta}|\underline{X}) \prod_j g(\beta_j|z_j) \quad (7)$$

or

$$p_{xyz}(\underline{\beta}|\underline{X},\underline{Y},\underline{Z}) \propto L_{xy}(\underline{\beta}|\underline{X},\underline{Y}) \prod_j g(\beta_j|z_j) \quad (8)$$

(Mislevy, in press). Standard Bayesian procedures for estimating item and population parameters that do not employ collateral information extend to (7) and (8) in a straightforward manner (Mislevy, 1987, in press).

Increase in Information: Theoretical Results

Using general results about missing data problems, such as Orchard and Woodbury's (1972) "missing information principle," it is possible to derive upper and lower bounds for the expected precision of item parameter estimates with and without collateral

information (Mislevy and Sheehan, 1988, in press). The results are expressed most easily in Bayesian terms.

Consider first the impact of collateral information about examinees. Let $V(\underline{\beta}|\underline{\theta},\underline{X},\underline{Y})$ represent the posterior variance of $\underline{\beta}$ that would be obtained after observing values of not only item responses x and collateral variables y from a sample of N examinees, but values of their latent proficiencies θ as well. Let analogous expressions represent posterior variance of $\underline{\beta}$ when values of one or more types of variables are not observed; for example, $V(\underline{\beta}|\underline{X})$ when only item responses are observed. The following relationships may be derived:

$$\begin{aligned} E[V(\underline{\beta}|\underline{\theta},\underline{X},\underline{Y})] &= E[V(\underline{\beta}|\underline{\theta},\underline{X})] \\ &\leq E[V(\underline{\beta}|\underline{X},\underline{Y})] \\ &\leq E[V(\underline{\beta}|\underline{X})] , \end{aligned}$$

where $\underline{A} \leq \underline{B}$ means that the matrix difference $\underline{B} - \underline{A}$ is at least positive semidefinite. Thus the precision of item parameter estimation when using collateral information about examinees along with item responses is at least as great as that expected when using item responses alone, but cannot exceed the precision that would be expected with the same sample size if values of the latent variable θ could be observed as well.

An obvious lower bound holds the impact of collateral information about items:

$$E[V(\underline{\beta}|\underline{X},\underline{Z})] \leq E[V(\underline{\beta}|\underline{X})] ;$$

that is, expected precision when using collateral information about items in addition to item responses, equals or exceeds precision expected when not using it. No ordering holds between $E[V(\underline{\beta}|\underline{X},\underline{Z})]$ and $E[V(\underline{\beta}|\underline{\theta},\underline{X})]$ in general. In particular, when \underline{Z} is employed along with \underline{X} , it is possible to exceed the precision obtainable with $\underline{\theta}$ and \underline{X} .

Increase in Information: Practical Results

By examining the structure of information matrices with and without collateral information, and by applying the methods to data from the National Assessment of Educational Progress (NAEP) and the Profile of American Youth surveys, it was found that modest increases in the precision of item parameter estimates can be achieved by using collateral information (Mislevy, 1987, in press; Mislevy and Sheehan, 1988, in press).

From collateral information about examinees, increases in information depend on the strength of the relationship of the collateral variables with θ . In typical educational and psychological settings where collateral information can often account for about a third of the population variance, and with item reliabilities typical of those settings, gains equivalent to 2 to 6 additional test items can be expected. This gain is substantial when few responses are available from each examinee, as in educational assessments, and may be useful in adaptive testing where tests are short but well-targeted. It is

unimpressive in individual achievement testing, where tests of sixty items or more are common.

From collateral information about items, increases equivalent to hundred and fifty additional examinees were found for Rasch item difficulty parameters in a junior high fractions test (Mislevy, in press). While a gain of this magnitude would be unimpressive in applications where data from thousands of examinees is already at hand, it is meaningful in situations when either (1) few examinees have been tested, as in the fractions example or in local testing problems, or (2) no examinees have been tested, as when approximating item statistics for newly-written test items.

In addition to small-sample applications, collateral information about items can play an important role in both item construction and diagnosis regardless of sample size. The conditional distributions of item parameters, $p(\beta|z)$, express item operating characteristics such as difficulty in terms of salient features of the items. To the degree that these distributions succeed in explaining item operating characteristics, the test constructor can manipulate the features to modify items in intended ways or to create new items that tap the same essential skills. To the degree that items depart from the centers of these predictive distributions, they are hard or easy for reasons other than those held most important in describing the domain. Outliers are suspect as flawed or irrelevant. The approach implied by (5) and (6) is a step in the direction of integrating educational and

psychological theory into the measurement process. (Its application to the items in the Document Utilization scale of the NAEP Survey of Adult Literacy is currently in progress.)

When Collateral Information Must Be Used

The preceding sections discuss how, when all examinees are presented all items, collateral information about examinees and items may be exploited to obtain more precise item parameter estimates. Consistent estimates are still obtained in this case if the collateral information is not used (Mislevy and Sheehan, in press). The same results apply when each examinee receives only a random subset of items.

This is not the case that obtains in many practical applications of IRT, however. In order to obtain more information about item or examinee parameters per observed response, items are often administered to examinees as a function of item and examinee collateral variables. Fourth grade students may be presented an easier test form than the overlapping form fifth graders receive, for example; and a high school graduate may be presented a harder item first in an adaptive test than a nongraduate. In order to obtain consistent MML item parameter estimates, it is mandatory to employ collateral information about examinees--i.e., to use (4) rather than (3) (Mislevy and Sheehan, in press). In order to obtain the correct Bayesian inferences, it is mandatory to use collateral information about items as well--i.e., to base inferences on (8) rather than (4) (Mislevy and Wu, 1988). Mislevy

and Sheehan (in press) give a simple counterexample with the Rasch model to demonstrate an asymptotic bias in item parameter estimation in such a case if collateral information is ignored.

Modeling Item Responses when Different Examinees Follow Different Solution Strategies

Initial work on using collateral information about items assumed that the IRT model was strictly correct. Thinking about the features of items that made them easy or hard, however, made it clear that difficulty depends on the way that the examinees are attempting to arrive at their answers. In particular, different features of items can make them differentially difficult for examinees who follow different solution strategies. This insight led to the formulation of a mixture of IRT models (Mislevy and Verhelst, in press). Resolving the mixture demands a type of collateral information that plays no role whatsoever in traditional psychometrics, including standard IRT: psychological theory about the different strategies that examinees might follow.

The key idea is to model item difficulty in terms of salient item features--features that tend to make an item easy or difficult under various strategies. The Mislevy-Verhelst model makes the following assumptions:

1. A finite number of known solution strategies apply.
2. Each examinee is applying the only one of these strategies for all the items in the set.

3. The responses of an examinee are observed but the strategy he or she has employed is not.
4. The responses of examinees following Strategy k conform to an item response model of a known form.
5. Substantive theory posits relationships between observable features of items and the probabilities of success enjoyed by members of each strategy class. The relationships may be known either fully or only partially--e.g., known as to parametric form but not parameter values.

Let $\theta = (\theta_1, \dots, \theta_K)$ be an examinee proficiency parameter, with the element θ_k corresponding to proficiency if Strategy k is employed. Let $\phi = (\phi_1, \dots, \phi_K)$ be an examinee strategy parameter, with all elements zero except for the single element k corresponding to the strategy that is employed; this element takes the value 1. Let the operating characteristics of Item j under Strategy k be given as follows:

$$P[x_j | \theta_k, \beta_k(z_{jk} | \alpha), \phi_k = 1] = f_k[x_j | \theta_k, \beta_k(z_{jk} | \alpha)] , \quad (9)$$

where $\beta_k(z_{jk} | \alpha)$, the item parameter for Item j that applies when examinees follow Strategy k, depends on its salient features z_{jk} under that strategy and a relatively small number of basic strategy parameters α . The MML function for estimating α induced by the data matrix \underline{X} from a sample of N examinees and the item/strategy collateral variables \underline{Z} is obtained as

$$L(\alpha|\underline{X}, \underline{Z}) = \prod_{i=1}^N \sum_{k=1}^K \pi_k \prod_{j=1}^n f_k[x_{ij}|\theta, \beta_k(z_{jk}|\alpha)] g_k(\theta) d\theta, \quad (10)$$

where g_k is the density of θ_k among those examinees following Strategy k , and π_k is the proportion of the population who do so. If the g_k s and the π s are not known, they too can be estimated via MML by maximizing (10) with respect to them as well.

If the α s, g_k s, and π s are known or well estimated, it is possible to calculate for a given examinee the probability that his response vector was produced under a given strategy and to estimate his ability under each possibility. By Bayes theorem, the posterior probability of Strategy k and proficiency θ under that strategy is obtained as

$$P(\theta, \phi_k=1|\underline{x}) = C f_k(\underline{x}|\theta, \beta_k(z_{jk})) g_k(\theta) \pi_k,$$

where C is the normalizing constant obtained as

$$C^{-1} = \sum_k \int f_k(\underline{x}|\theta, \beta_k(z_{jk})) g_k(\theta) d\theta \pi_k.$$

The posterior probability that Strategy k was employed is

$$P(\phi_k=1|\underline{x}) = \int P(\theta, \phi_k=1|\underline{x}) d\theta$$

and the posterior mean proficiency conditional on $\phi_k=1$ (i.e., supposing that Strategy k was used) is

$$E(\theta_k | \underline{x}, \phi_k = 1) = \int \theta P(\theta, \phi_k = 1 | \underline{x}) d\theta P^{-1}(\phi_k = 1 | \underline{x}) .$$

The significance of this model lies in its ability to express how examinees solve items rather than just how many they solve.

The latter is all that the standard models of test theory can do. Areas of potential benefit include psychological investigations of alternative processing models, educational decisions involving level of understanding, and determinations of alternative mental models in problem solving. The approach opens the door to such applications as (1) adaptive testing schemes designed to infer how examinees solve problems as well as how well they solve them, and (2) studies of changes in the structure as well as the level of intelligence in the course of human development.

Inferring Examinee Ability When Some Item Responses Are Missing

In practical applications of item response theory (IRT), there are several reasons that item responses may not be observed from all examinees to all test items. The reason most germane to the collateral information problem is the intentional administration of only subsets of items to examinees, with the subset depending on collateral information. It was mentioned above that collateral information must be taken into account in these cases. In addition to this type of missingness, Mislevy and Wu (1988) studied problems of inference that arise with several other types of missingness that arise frequently in IRT.

To preface the results of their study, we review Rubin's (1976) notions about "ignorability" of missing data. Ignoring the

missingness process under direct likelihood inference means using a pseudo-likelihood that includes terms for only the responses that were observed, without regard for the processes by which they came to be observed. The resulting inferences are appropriate if the pseudo-likelihood is proportional to the correct likelihood that does account for the missingness process. In this case the correct point estimate of the maximum likelihood estimate (MLE) is obtained. Sampling-distribution inferences based on the MLE are appropriate only if the missingness pattern does not depend on the values of the observed data. When this condition holds, sampling-distribution inferences can be drawn with regard to repeated samples of responses to only those items whose responses were observed. The missingness process is ignorable with respect to Bayesian inference if the correct Bayesian posterior is proportional to the product of the pseudo-likelihood and an appropriate prior distribution.

For five common types of missingness in IRT, Mislevy and Wu first used Rubin's (1976) theorems to determine whether ignorability holds under direct likelihood and Bayesian inference about examinee parameters θ when item parameters β are known. In those cases in which the correct value of the MLE is obtained under direct likelihood inference, they asked whether sampling distribution inferences based on the MLE were appropriate. They then considered the analogous questions for inferences about β when the examinee parameters are eliminated by marginalization, as

in (3)-(8). The findings are summarized below. Tables 2 and 3 highlight the results on ignorability.

Tables 2 and 3 about here

Case 1: Alternate Test Forms. When an examinee is assigned one of several alternative test forms by a random process such as a coin flip or a spiralling scheme, the process that renders missing the responses to items on the forms not presented is ignorable for all three types of inference, both for estimating β and for estimating θ when β is known.

Case 2: Targeted Testing. When collateral variables such as educational or demographic status are used to assign an examinee one of several test forms that differ in their measurement properties, the resulting missingness on forms not given is ignorable under direct likelihood inference for θ given β , but not under Bayesian inference unless the prior information about examinees that led to differential assignments is conditioned on. This information must be taken into account for both likelihood and Bayesian inferences about β ; for Bayesian inference, prior information about β used to select items must additionally be taken into account. Sampling distribution inferences may be based on MLEs for β and for θ given β , conditional on the observed patterns of form administration within values of the examinee variables used for targeting.

It should be emphasized that these conclusions depend on the veracity of the IRT model. In particular, it is necessary that the regression of a correct response on ability be invariant with respect to collateral information. This assumption may well fail in a situation of currently increasing interest: An item pool is calibrated using an IRT model, and a school is allowed to measure students using only those items it deems relevant to its curriculum. If students from different schools have had different opportunities to learn the skills tapped by different items, then tailoring tests to their strengths leads almost certainly to item by school by ability interactions--a violation of the IRT model. Estimates for schools and individuals within schools tend to overestimate the scores they would have received had they been given all items, or randomly selected subsets of items. This use of IRT may hold practical value nonetheless, provided that such scores are viewed not as consistent estimates of performance in the total pool but as indicators of a kind of maximal performance.

Case 3: Adaptive Testing. In adaptive testing, item assignment proceeds item by item for each examinee according to the values of his responses to preceding items. The same conclusions as for Case 2 hold for direct likelihood and Bayesian inference. Ignorability under direct likelihood inference means that the correct points are identified as MLEs of θ given $\underline{\beta}$ and of $\underline{\beta}$. The usual MLE properties under sampling-distribution inference need not hold, however, because the probabilities of missingness patterns depend on the values of observed responses.

Case 4: Not-reached Items. When some examinees run out of time before they see the last items on a nearly nonspeeded test, the not-reached process is ignorable with respect to direct likelihood inference about θ given $\underline{\beta}$, and the MLE supports sampling distribution inferences that pertain to repeated administrations of the items that were actually reached. This missingness process is not ignorable under Bayesian inference unless speed and ability are independent. And only then can direct likelihood inferences about $\underline{\beta}$ ignore the missingness. Furthermore, Bayesian inferences about $\underline{\beta}$ require that collateral variables for items be employed if they played a role in determining which items would not be reached, as when items are ordered from easy to hard.

Case 5: Intentional Omission. When examinees are presented items, have a chance to appraise their content, and decide for their own reasons not to respond, the missingness is not ignorable. Inferences must be drawn from a full model for the joint distribution of missingness and item response.

Not surprisingly, modeling this nonignorable nonresponse is difficult. Neither of the two most ambitious approaches proposed to date, namely Lord's (1983) model for omits and the use of multiple-category IRT models (e.g., Bock, 1972), handles the issue of local independence in a fully satisfactory manner. Under Lord's (1983) model, the marginal model for item responses is not a standard IRT model depending on θ alone and exhibiting local independence. Under the multiple-category model approach, local

independence fails unless all examinees at any given ability level have the same propensity to omit items they are unsure of, rather than guess at random.

If one assumes that examinees are perfect judges of their chances of responding correctly, and omit only if it is in accordance with the strategy that maximizes their expected score, Lord's (1974) treatment of omits as fractionally correct can be justified as providing the expectation of a conditional term in the full likelihood for omission probabilities and correct-response probabilities. This procedure is readily incorporated into standard complete-data IRT algorithms and avoids having to specify the full likelihood, but sacrifices information about examinee and item parameters conveyed by the observed pattern of missingness. Given the complexity of models for the full likelihood, however, this expedient seems to be a good practical choice--provided that, as Lord urges, examinees are clearly informed about how omits will be scored and which omitting strategy maximizes their chances of scoring well.

Conclusion

Although collateral information about examinees and items is rarely employed in item response theory (IRT), it is straightforward to incorporate it using Bayesian and empirical Bayesian methods. If the IRT model is correct and examinees are assigned items independently of values on collateral variables, then collateral information can be used to improve item parameter estimation modestly. Employing collateral information is

mandatory to obtain correct Bayesian and empirical Bayesian inferences if it was used to assign items to examinees.

Aside from considerations of efficiency, employing collateral information about items is a step toward integrating educational and psychological theory into the measurement process. Two aspects of this idea were developed in the course of the project.

The first, which takes a more traditional measurement perspective, assumes that a single IRT model provides an acceptable fit to the data of interest. Modeling items' operating characteristics in terms of salient features can make estimation more precise, but more importantly it elucidates the reasons that items are hard or easy, and why some are more discriminating than others. A formal framework is thus available for item construction and diagnosis, expressing relationships among substantive theory, item features, and measurement properties.

The second is a response to a growing awareness of the fact that traditional psychometric models (IRT as well as classical test theory) measure what is essentially an overall level of proficiency--losing in the process qualitative differences among examinees that arise from different cognitive solution strategies. In order to extend psychometric analysis to these problems, and to bring to bear the findings of recent research upon applied measurement problems, it is mandatory to employ collateral information about examinees and items that bears upon the ways that people solve problems. A mixture of IRT models that applies to some problems of this type was introduced in the project.

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Table 1

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Table 2
Ignorability Results for Estimating θ Given β

Type of Missingness	Type of Inference		
	Direct Likelihood	Bayesian	Sampling Distribution*
Alternate Forms	Yes	Yes	Yes
Targeted Forms	Yes	Yes, given examinee variables	Yes
Adaptive Testing	Yes	Yes, given examinee variables if they are used	No
Not-Reached	Yes	No, unless speed and ability are independent	Yes
Intentional Omissions	No	No	No

* Conditional on the observed pattern of missingness.

Table 3

Ignorability Results for Estimating β After Marginalizing over θ

Type of Missingness	Type of Inference		
	Direct Likelihood	Bayesian	Sampling Distribution*
Alternate Forms	Yes	Yes	Yes
Targeted Forms	Yes, given examinee variables	Yes, given examinee and item variables	Yes, given examinee variables
Adaptive Testing	Yes, given examinee variables if they are used	Yes, given item variables and examinee variables if they are used	No
Not-Reached	No, unless speed and ability are independent	No, unless speed and ability are independent	No, unless speed and ability are independent
Intentional Omissions	No	No	No

* Conditional on the observed pattern of missingness.

Educational Testing Service/Mislevy

Dr. Terry Ackerman
American College Testing Programs
P.O. Box 168
Iowa City, IA 52243

Dr. Robert Ahlers
Code N711
Human Factors Laboratory
Naval Training Systems Center
Orlando, FL 32813

Dr. James Algina
1403 Norman Hall
University of Florida
Gainesville, FL 32605

Dr. Erling B. Andersen
Department of Statistics
Studiestraede 6
1455 Copenhagen
DENMARK

Dr. Eva L. Baker
UCLA Center for the Study
of Evaluation
145 Moore Hall
University of California
Los Angeles, CA 90024

Dr. Isaac Bejar
Mail Stop: 10-R
Educational Testing Service
Rosedale Road
Princeton, NJ 08541

Dr. Menucha Birenbaum
School of Education
Tel Aviv University
Ramat Aviv 69978
ISRAEL

Dr. Arthur S. Blaiwes
Code N712
Naval Training Systems Center
Orlando, FL 32813-7100

Dr. Bruce Bloxom
Defense Manpower Data Center
550 Camino El Estero,
Suite 200
Monterey, CA 93943-3231

Dr. R. Darrell Bock
University of Chicago
NORC
6030 South Ellis
Chicago, IL 60637

Cdt. Arnold Bohrer
Sectie Psychologisch Onderzoek
Rekruterings-En Selectiecentrum
Kwartier Koningen Astrid
Bruijnstraat
1120 Brussels, BELGIUM

Dr. Robert Breaux
Code 7B
Naval Training Systems Center
Orlando, FL 32813-7100

Dr. Robert Brennan
American College Testing
Programs
P. O. Box 168
Iowa City, IA 52243

Dr. James Carlson
American College Testing
Program
P.O. Box 168
Iowa City, IA 52243

Dr. John B. Carroll
409 Elliott Rd., North
Chapel Hill, NC 27514

Dr. Robert M. Carroll
Chief of Naval Operations
OP-0182
Washington, DC 20350

Dr. Raymond E. Christal
UES LAMP Science Advisor
AFHRL/MOEL
Brooks AFB, TX 78235

Dr. Norman Cliff
Department of Psychology
Univ. of So. California
Los Angeles, CA 90089-1061

Educational Testing Service/Mislevy

Director,
Manpower Support and
Readiness Program
Center for Naval Analysis
2000 North Beauregard Street
Alexandria, VA 22311

Dr. Stanley Collier
Office of Naval Technology
Code 222
800 N. Quincy Street
Arlington, VA 22217-5000

Dr. Hans F. Crombag
Faculty of Law
University of Limburg
P.O. Box 616
Maastricht
The NETHERLANDS 6200 MD

Dr. Timothy Davey
Educational Testing Service
Princeton, NJ 08541

Dr. C. M. Dayton
Department of Measurement
Statistics & Evaluation
College of Education
University of Maryland
College Park, MD 20742

Dr. Ralph J. DeAyala
Measurement, Statistics,
and Evaluation
Benjamin Bldg., Rm. 4112
University of Maryland
College Park, MD 20742

Dr. Dattprasad Divgi
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. Hei-Ki Dong
Bell Communications Research
6 Corporate Place
PYA-1K226
Piscataway, NJ 08854

Dr. Fritz Drasgow
University of Illinois
Department of Psychology
603 E. Daniel St.
Champaign, IL 61820

Defense Technical
Information Center
Cameron Station, Bldg 5
Alexandria, VA 22314
Attn: TC
(12 Copies)

Dr. Stephen Dunbar
224B Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. James A. Earles
Air Force Human Resources Lab
Brooks AFB, TX 78235

Dr. Kent Eaton
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. John M. Eddins
University of Illinois
252 Engineering Research
Laboratory
103 South Mathews Street
Urbana, IL 61801

Dr. Susan Embretson
University of Kansas
Psychology Department
426 Fraser
Lawrence, KS 66045

Dr. George Englehard, Jr.
Division of Educational Studies
Emory University
210 Fishburne Bldg.
Atlanta, GA 30322

Dr. Benjamin A. Fairbank
Performance Metrics, Inc.
5825 Callaghan
Suite 225
San Antonio, TX 78228

Educational Testing Service/Mislevy

Dr. P-A. Federico
Code 51
NPRDC
San Diego, CA 92152-6800

Dr. Leonard Feldt
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. Richard L. Ferguson
American College Testing
P.O. Box 168
Iowa City, IA 52243

Dr. Gerhard Fischer
Liebiggasse 5/3
A 1010 Vienna
AUSTRIA

Dr. Myron Fischl
U.S. Army Headquarters
DAPE-MRR
The Pentagon
Washington, DC 20310-0300

Prof. Donald Fitzgerald
University of New England
Department of Psychology
Armidale, New South Wales 2351
AUSTRALIA

Mr. Paul Foley
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Alfred R. Fregly
AFOSR/NL, Bldg. 410
Bolling AFB, DC 20332-6448

Dr. Robert D. Gibbons
Illinois State Psychiatric Inst.
Rm 529W
1601 W. Taylor Street
Chicago, IL 60612

Dr. Janice Gifford
University of Massachusetts
School of Education
Amherst, MA 01003

Dr. Robert Glaser
Learning Research
& Development Center
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 15260

Dr. Bert Green
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

DORNIER GMBH
P.O. Box 1420
D-7990 Friedrichshafen 1
WEST GERMANY

Dr. Ronald K. Hambleton
University of Massachusetts
Laboratory of Psychometric
and Evaluative Research
Hills South, Room 152
Amherst, MA 01003

Dr. Delwyn Harnisch
University of Illinois
51 Gerty Drive
Champaign, IL 61820

Dr. Grant Henning
Senior Research Scientist
Division of Measurement
Research and Services
Educational Testing Service
Princeton, NJ 08541

Ms. Rebecca Hetter
Navy Personnel R&D Center
Code 63
San Diego, CA 92152-6800

Dr. Paul W. Holland
Educational Testing Service, 21-T
Rosedale Road
Princeton, NJ 08541

Prof. Lutz F. Hornke
Institut fur Psychologie
RWTH Aachen
Jaegerstrasse 17/19
D-5100 Aachen
WEST GERMANY

Educational Testing Service/Mislevy

Dr. Paul Horst
677 G Street, #184
Chula Vista, CA 92010

Mr. Dick Hoshaw
OP-135
Arlington Annex
Room 2834
Washington, DC 20350

Dr. Lloyd Humphreys
University of Illinois
Department of Psychology
603 East Daniel Street
Champaign, IL 61820

Dr. Steven Hunka
3-104 Educ. N.
University of Alberta
Edmonton, Alberta
CANADA T6G 2G5

Dr. Huynh Huynh
College of Education
Univ. of South Carolina
Columbia, SC 29208

Dr. Robert Jannarone
Elec. and Computer Eng. Dept.
University of South Carolina
Columbia, SC 29208

Dr. Douglas H. Jones
Thatcher Jones Associates
P.O. Box 6640
10 Trafalgar Court
Lawrenceville, NJ 08648

Dr. Milton S. Katz
European Science Coordination
Office
U.S. Army Research Institute
Box 65
FPO New York 09510-1500

Prof. John A. Keats
Department of Psychology
University of Newcastle
N.S.W. 2308
AUSTRALIA

Dr. G. Gage Kingsbury
Portland Public Schools
Research and Evaluation Department
501 North Dixon Street
P. O. Box 3107
Portland, OR 97209-3107

Dr. William Koch
Box 7246, Meas. and Eval. Ctr.
University of Texas-Austin
Austin, TX 78703

Dr. James Kraatz
Computer-based Education
Research Laboratory
University of Illinois
Urbana, IL 61801

Dr. Leonard Kroeker
Navy Personnel R&D Center
Code 62
San Diego, CA 92152-6800

Dr. Jerry Lehnus
Defense Manpower Data Center
Suite 400
1600 Wilson Blvd
Rosslyn, VA 22209

Dr. Thomas Leonard
University of Wisconsin
Department of Statistics
1210 West Dayton Street
Madison, WI 53705

Dr. Michael Levine
Educational Psychology
210 Education Bldg.
University of Illinois
Champaign, IL 61801

Dr. Charles Lewis
Educational Testing Service
Princeton, NJ 08541-0001

Dr. Robert L. Linn
Campus Box 249
University of Colorado
Boulder, CO 80309-0249

Educational Testing Service/Mislevy

Dr. Robert Lockman
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. Frederic M. Lord
Educational Testing Service
Princeton, NJ 08541

Dr. George B. Macready
Department of Measurement
Statistics & Evaluation
College of Education
University of Maryland
College Park, MD 20742

Dr. Gary Marco
Stop 31-E
Educational Testing Service
Princeton, NJ 08451

Dr. James R. McBride
The Psychological Corporation
1250 Sixth Avenue
San Diego, CA 92101

Dr. Clarence C. McCormick
HQ, USMEPCOM/MEPCT
2500 Green Bay Road
North Chicago, IL 60064

Dr. Robert McKinley
Educational Testing Service
16-T
Princeton, NJ 08541

Dr. James McMichael
Technical Director
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Barbara Means
SRI International
333 Ravenswood Avenue
Menlo Park, CA 94025

Dr. Robert Mislevy
Educational Testing Service
Princeton, NJ 08541

Dr. William Montague
NPRDC Code 13
San Diego, CA 92152-6800

Ms. Kathleen Moreno
Navy Personnel R&D Center
Code 62
San Diego, CA 92152-6800

Headquarters Marine Corps
Code MPI-20
Washington, DC 20380

Dr. W. Alan Nicewander
University of Oklahoma
Department of Psychology
Norman, OK 73071

Deputy Technical Director
NPRDC Code 01A
San Diego, CA 92152-6800

Director, Training Laboratory,
NPRDC (Code 05)
San Diego, CA 92152-6800

Director, Manpower and Personnel
Laboratory,
NPRDC (Code 06)
San Diego, CA 92152-6800

Director, Human Factors
& Organizational Systems Lab,
NPRDC (Code 07)
San Diego, CA 92152-6800

Library, NPRDC
Code P201L
San Diego, CA 92152-6800

Commanding Officer,
Naval Research Laboratory
Code 2627
Washington, DC 20390

Dr. Harold F. O'Neil, Jr.
School of Education - WPH 801
Department of Educational
Psychology & Technology
University of Southern California
Los Angeles, CA 90089-0031

Educational Testing Service/Mislevy

Dr. James B. Olsen
WICAT Systems
1875 South State Street
Orem, UT 84058

Office of Naval Research,
Code 1142CS
800 N. Quincy Street
Arlington, VA 22217-5000
(6 Copies)

Office of Naval Research,
Code 125
800 N. Quincy Street
Arlington, VA 22217-5000

Assistant for MPT Research,
Development and Studies
OP 01B7
Washington, DC 20370

Dr. Judith Orasanu
Basic Research Office
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. Jesse Orlansky
Institute for Defense Analyses
1801 N. Beauregard St.
Alexandria, VA 22311

Dr. Randolph Park
Army Research Institute
5001 Eisenhower Blvd.
Alexandria, VA 22333

Wayne M. Patience
American Council on Education
GED Testing Service, Suite 20
One Dupont Circle, NW
Washington, DC 20036

Dr. James Paulson
Department of Psychology
Portland State University
P.O. Box 751
Portland, OR 97207

Dept. of Administrative Sciences
Code 54
Naval Postgraduate School
Monterey, CA 93943-5026

Department of Operations Research,
Naval Postgraduate School
Monterey, CA 93940

Dr. Mark D. Reckase
ACT
P. O. Box 168
Iowa City, IA 52243

Dr. Malcolm Ree
AFHRL/MOA
Brooks AFB, TX 78235

Dr. Barry Riegelhaupt
HumRRO
1100 South Washington Street
Alexandria, VA 22314

Dr. Carl Ross
CNET-POCD
Building 90
Great Lakes NTC, IL 60088

Dr. J. Ryan
Department of Education
University of South Carolina
Columbia, SC 29208

Dr. Fumiko Samejima
Department of Psychology
University of Tennessee
310B Austin Peay Bldg.
Knoxville, TN 37916-0900

Mr. Drew Sands
NPRDC Code 62
San Diego, CA 92152-6800

Lowell Schoer
Psychological & Quantitative
Foundations
College of Education
University of Iowa
Iowa City, IA 52242

Dr. Mary Schratz
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Dan Segall
Navy Personnel R&D Center
San Diego, CA 92152

Educational Testing Service/Mislevy

Dr. W. Steve Sellman
OASD (MRA&L)
28269 The Pentagon
Washington, DC 20301

Dr. Kazuo Shigemasa
7-9-24 Kugenuma-Kaigan
Fujisawa 251
JAPAN

Dr. William Sims
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. H. Wallace Sinaiko
Manpower Research
and Advisory Services
Smithsonian Institution
801 North Pitt Street, Suite 120
Alexandria, VA 22314-1713

Dr. Richard E. Snow
School of Education
Stanford University
Stanford, CA 94305

Dr. Richard C. Sorensen
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Paul Speckman
University of Missouri
Department of Statistics
Columbia, MO 65201

Dr. Judy Spray
ACT
P.O. Box 168
Iowa City, IA 52243

Dr. Martha Stocking
Educational Testing Service
Princeton, NJ 08541

Dr. William Stout
University of Illinois
Department of Statistics
101 Illini Hall
725 South Wright St.
Champaign, IL 61820

Dr. Hariharan Swaminathan
Laboratory of Psychometric and
Evaluation Research
School of Education
University of Massachusetts
Amherst, MA 01003

Mr. Brad Sympson
Navy Personnel R&D Center
Code-62
San Diego, CA 92152-6800

Dr. John Tangney
AFOSR/NL, Bldg. 410
Bolling AFB, DC 20332-6448

Dr. Kikumi Tatsuoka
CERL
252 Engineering Research
Laboratory
103 S. Mathews Avenue
Urbana, IL 61801

Dr. Maurice Tatsuoka
220 Education Bldg
1310 S. Sixth St.
Champaign, IL 61820

Dr. David Thissen
Department of Psychology
University of Kansas
Lawrence, KS 66044

Mr. Gary Thomasson
University of Illinois
Educational Psychology
Champaign, IL 61820

Dr. Robert Tsutakawa
University of Missouri
Department of Statistics
222 Math. Sciences Bldg.
Columbia, MO 65211

Dr. Ledyard Tucker
University of Illinois
Department of Psychology
603 E. Daniel Street
Champaign, IL 61820

1988/07/06

Educational Testing Service/Mislevy

Dr. Vern W. Urry
Personnel R&D Center
Office of Personnel Management
1900 E. Street, NW
Washington, DC 20415

Dr. David Vale
Assessment Systems Corp.
2233 University Avenue
Suite 440
St. Paul, MN 55114

Dr. Frank L. Vicino
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Howard Wainer
Educational Testing Service
Princeton, NJ 08541

Dr. Ming-Mei Wang
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. Thomas A. Warm
Coast Guard Institute
P. O. Substation 18
Oklahoma City, OK 73169

Dr. Brian Waters
HumRRO
12908 Argyle Circle
Alexandria, VA 22314

Dr. David J. Weiss
N660 Elliott Hall
University of Minnesota
75 E. River Road
Minneapolis, MN 55455-0344

Dr. Ronald A. Weitzman
Box 146
Carmel, CA 93921

Major John Welsh
AFHRL/MOAN
Brooks AFB, TX 78223

Dr. Douglas Wetzel
Code 51
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Rand R. Wilcox
University of Southern
California
Department of Psychology
Los Angeles, CA 90089-1061

German Military Representative
ATTN: Wolfgang Wildgrube
Streitkraefteamt
D-5300 Bonn 2
4000 Brandywine Street, NW
Washington, DC 20016

Dr. Bruce Williams
Department of Educational
Psychology
University of Illinois
Urbana, IL 61801

Dr. Hilda Wing
NRC MH-176
2101 Constitution Ave.
Washington, DC 20418

Dr. Martin F. Wiskoff
Defense Manpower Data Center
550 Camino El Estero
Suite 200
Monterey, CA 93943-3231

Mr. John H. Wolfe
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. George Wong
Biostatistics Laboratory
Memorial Sloan-Kettering
Cancer Center
1275 York Avenue
New York, NY 10021

Dr. Wallace Wulfbeck, III
Navy Personnel R&D Center
Code 51
San Diego, CA 92152-6800

1988/07/06

Educational Testing Service/Mislevy

Dr. Kentaro Yamamoto
03-T
Educational Testing Service
Rosedale Road
Princeton, NJ 08541

Dr. Wendy Yen
CTB/McGraw Hill
Del Monte Research Park
Monterey, CA 93940

Dr. Joseph L. Young
National Science Foundation
Room 320
1800 G Street, N.W.
Washington, DC 20550

Mr. Anthony R. Zara
National Council of State
Boards of Nursing, Inc.
625 North Michigan Avenue
Suite 1544
Chicago, IL 60611

Dr. Peter Stoloff
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268